MASS TRANSFER IN A LIQUID LAYER OF VARIABLE THICKNESS ON A ROTATING ARCHIMEDES SPIRAL TAKING ACCOUNT OF THE ENTRANCE REGION

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In [1] the method of integral relations was used to investigate the hydrodynamics and mass transfer in a liquid layer on a rotating Archimedes spiral in the absence of wave formation. In the present article we use the work method [2, 3] to investigate the hydrodynamics and mass transfer in the entrance region in a liquid layer of variable thickness on a rotating Archimedes spiral.

We consider the flow of a laminar liquid layer on the inner surface of an Archimedes spiral rotating in the horizontal plane with a constant angular velocity ω. The equation of the spiral in polar coordinates is $r = A\theta$, where $A > 0$. The liquid film flows from the center (entrance region) toward the periphery in the channel formed by the Archimedes spiral. We denote the arc length along the streamlined wall of the spiral channel from the origin of coordinates (x, y) in the plane of the inlet by x, and the perpendicular distance from the wall by y. The coordinate system is fixed with respect to the streamlined solid surface. It is assumed that the pressure is steady, the flows are isothermal, and the diffusion coefficient is constant.

Under these assumptions the problem is described by the Prandtl boundary-layer equations and the equation of convective diffusion,

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=F_x-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2},\quad -\frac{u^2}{R(x)}=F_y-\frac{1}{\rho}\frac{\partial p}{\partial y},\quad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0; \tag{1}$$

$$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D\frac{\partial^2 c}{\partial y^2}, \tag{2}$$

where D is the diffusion coefficient, $R(x) = A(\theta^2+1)^{3/2}/(\theta^2+2)$ is the radius of curvature of the spiral, and F_x and F_y are the components of the body forces along the coordinate axes. The body forces acting on a unit mass of the liquid film are the centrifugal force $F_{ce} = \omega^2 R(x)$ and the Coriolis force $F_{co} = 2[\omega \times v]$. Their components along the coordinate axes have the form

$$F_x = \omega^2 R(x)\cos\alpha \pm 2\omega v,\quad F_y = -\omega^2 R(x)\sin\alpha \mp 2\omega u,$$

where the upper signs correspond to a counterclockwise rotation of the spiral and the lower to clockwise rotation. The angle α between the centrifugal force vector and the positive direction of the tangent is related to the polar angle θ by the equations

$$\sin\alpha = \theta/\sqrt{\theta^2+1},\quad \cos\alpha = 1/\sqrt{\theta^2+1}.$$

We use the following boundary and initial conditions:

$$\begin{aligned}&\text{for}\quad y=0\quad u=0,\ v=0,\ c=0,\\ &\text{for}\quad y=H(x),\ \partial u/\partial y=0,\ p=\text{const},\ c=c_p,\\ &\text{for}\quad x=0,\ c=0,\end{aligned}$$

where the equation of the surface H(x) is determined from the solution of Eqs. (1) and (2), taking account of the kinematic condition

$$v = u\,dH/dx$$

on the boundary surface.

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We construct dimensionless equations from (1) and (2) by introducing the following transformations:

$$u = u_p\bar{u},\ y = \delta_p\bar{y},\ v = (u_p/\mathrm{Re})\bar{v},\ x = \delta_p \mathrm{Re}\bar{x},\ c = c_p c,$$

where the film thickness δ_p in the stabilized region, found from the solution of Eqs. (1), has the form

$$\delta_p = \sqrt[3]{\frac{3q\nu}{\omega^2 A}}.$$

In the dimensionless equations we transform from the variables $(\bar{x}, \bar{y})$ to $(\bar{\theta}, \bar{y})$ with $d\bar{x} = \sqrt{\theta^2+1}\, d\theta/\mathrm{E1\ E5\ Re}$, where $\mathrm{E1} = \sqrt[3]{\mathrm{Re}/\mathrm{Ga}} = \delta_p/h_0$ is the ratio of the thickness of the boundary layer to the original thickness of the liquid film h_0, $\mathrm{Ga} = \omega^2 A h_0^3/\nu^2$ is the Galileo number, $\mathrm{Re} = 3q/\nu$ is the modified Reynolds number (q is the flow rate of the liquid film), and $\mathrm{E5} = h_0/A$ is the dimensionless characteristic of the spiral. Then the system of equations and the boundary conditions in the coordinates $(\theta, \bar{y})$ describing the motion of a thin layer on the inner surface of the spiral channel take the form (omitting bars over symbols)

$$\frac{\mathrm{E5\,E1\,Re}}{\sqrt{\theta^2+1}}\, u\frac{\partial u}{\partial\theta} + v\frac{\partial u}{\partial y} = F_x - \frac{\mathrm{E5\,E1\,Re}}{\sqrt{\theta^2+1}}\frac{\partial p}{\partial\theta} + 3\frac{\partial^2 u}{\partial y^2},$$
$$-\mathrm{E5\,E1}\frac{u^2}{R(x)} = F_y - \frac{\partial p}{\partial y},\quad \frac{\mathrm{E5\,E1\,Re}}{\sqrt{\theta^2+1}}\frac{\partial u}{\partial\theta} + \frac{\partial v}{\partial y} = 0, \tag{3}$$

$$F_x = 9\frac{\theta^2+1}{\theta^2+2} \pm \frac{6\,\mathrm{Ga}^{1/2}\,\mathrm{E5}^{1/2}\,\mathrm{E1}^2}{\mathrm{Re}}\, v,$$
$$F_y = -\frac{9}{\mathrm{Re}}\,\theta\frac{\theta^2+1}{\theta^2+2} \mp \frac{6\,\mathrm{Ga}^{1/2}\,\mathrm{E5}^{1/2}\,\mathrm{E1}^2}{\mathrm{Re}}\, u; \tag{4}$$

$$\frac{\mathrm{E5\,E1\,Re}}{\sqrt{\theta^2+1}}\, u\frac{\partial c}{\partial\theta} + v\frac{\partial c}{\partial y} = \frac{3}{\mathrm{Pr}}\frac{\partial^2 c}{\partial y^2};$$

$$\text{for}\quad y = 0\quad u = v = 0,\quad c = 0,$$
$$\text{for}\quad y = \frac{H(x)}{\delta_p}\quad \frac{\partial u}{\partial y} = 0,\quad \frac{\partial p}{\partial\theta} = 0,\quad c = 1. \tag{5}$$

We solve the problem by the method of equal flow-rate surfaces, which is related to the group of collocation methods [2]. We introduce the lines $y_k = y_k(x)$ into the flow field, and the notation

$$u_k(x) = u[x,\ y_k(x)];\ v_k(x) = v[x,\ y_k(x)];\ c_k(x) = c[x,\ y_k(x)].$$

We set ourselves the goal of reducing the problem of flow development to the numerical determination of the functions $u_k(x)$, $v_k(x)$, and the interphase surface $H(x)$. We define y_k as a line of equal flow rate; then $u_k(x)$ and $v_k(x)$ are connected by the relation

$$v_k = \frac{\mathrm{E5\,E1\,Re}}{\sqrt{\theta^2+1}}\, u_k(x)\frac{dy_k}{d\theta}, \tag{6}$$

which follows from the conservation of flow rate and the equation of continuity. In addition, the equation of continuity is equivalent to the following system of equations:

$$\int\limits_{y_{k-1}(x)}^{y_k(x)} u\,dy = \mathrm{const}_k,\quad k = 2, 3, \ldots, N.$$

Evaluating these integrals by the trapezoidal rule with a uniform error estimate with respect to the variable θ having the third order of smallness $O_3 = \{\max_k [y_k - y_{k-1}]\}^3$, we obtain a system of linear algebraic equations of the form

$$(y_k(x) - y_{k-1}(x)(u_k(x) + u_{k-1}(x)) = \mathrm{const}_k + O_3. \tag{7}$$

After differentiating both sides of (7) with respect to θ we obtain a system of ordinary differential equations for determining the surfaces of equal flow rate $y_k(x)$:

$$\frac{dy_k}{d\theta} = \frac{dy_{k-1}}{d\theta} - \frac{y_k - y_{k-1}}{u_k + u_{k-1}}\left(\frac{du_k}{d\theta} + \frac{du_{k-1}}{d\theta}\right). \tag{8}$$

The derivatives with respect to the independent variable θ have the form

$$\frac{d\varphi_k}{d\theta} = \left[\frac{\partial\varphi}{\partial\theta} + \frac{\partial\varphi}{\partial y}\frac{dy}{d\theta}\right]_{y=y_k}, \tag{9}$$

where $\varphi_k = u_k, p_k, c_k$.

Expressing the $\partial\varphi_k/\partial\theta$ from (9), substituting them into Eqs. (3) and (4), and using (6), we reduce the problem to the following system of ordinary differential equations:

$$\frac{\mathrm{E5\,E1\,Re}}{\sqrt{\theta^2+1}} u_k \frac{du_k}{d\theta} = F_{xk} - \frac{\mathrm{E5\,E1\,Re}}{\sqrt{\theta^2+1}} \left[\frac{dp_k}{d\theta} - \frac{\partial p_k}{\partial y_k}\frac{dy_k}{d\theta}\right] + 3\frac{\partial^2 u_k}{\partial y_k^2}; \tag{10}$$

$$\frac{dp_k}{d\theta} = \frac{dp_{k-1}}{d\theta} + \frac{dM_k}{d\theta}; \tag{11}$$

$$\frac{\mathrm{E5\,E1\,Re}}{\sqrt{\theta^2+1}} u_k \frac{dc_k}{d\theta} = \frac{3}{\mathrm{Pr}}\frac{\partial^2 c_k}{\partial y_k^2}, \tag{12}$$

where

$$M_k(\theta) = \int\limits_{y_{k-1}}^{y_k} \left(\mathrm{E5\,E1}\frac{u^2}{R(\theta)} + F_y\right) dy;$$

$$\frac{\partial p_k}{\partial y_k} = F_{yk} + \mathrm{E5\,E1}\frac{u_k^2}{R(\theta)}, \quad k = 2, 3, \ldots, N.$$

To evaluate the second derivatives with respect to y in Eqs. (10) and (12) we write the solutions for u_k and c_k as expansions in a complete set of basis functions satisfying the homogeneous boundary conditions (5). In the present paper the systems of basis functions for velocity and concentration were chosen, respectively, in the form

$$V_{kj}(x) = \left(\frac{j+1}{j} - \eta_k\right)\eta_k^j, \quad K1_{kj}(x) = \frac{1}{2}\left(\eta_k^j - \eta_k^{j+1}\right) + \eta_k^{j+1},$$

and the orthogonal Chebyshev polynomials of the first kind were used in the form

$$V_{kj}(x) = T_{j+1}(\eta_k) - T_{j+1}(0) - [T_j(\eta_k) - T_j(0)]\left(\frac{j+1}{j}\right)^2,$$
$$K1_{kj}(x) = T_{j+1}(\eta_k) + T_{j+1}(0)\left(2\eta_k^2 - \eta_k - 1\right),$$

where everywhere in these expressions

$$\eta_k(x) = y_k(x)/H(x), \; j = 1, \ldots, N, \; k = 1, \ldots, N.$$

The results obtained by using different sets of basis functions did not differ appreciably, but the required accuracy was attained with fewer terms when using the Chebyshev polynomials.

The solutions for $u_k(x)$ and $c_k(x)$ were written as

$$u_k(x) = \sum_{j=1}^{N} A_j(x) V_{kj}(x),$$
$$c_k(x) = \sum_{j=1}^{N} A1_j(x) K1_{kj}(x),$$
$$j = 1, \ldots, N, \; k = 1, \ldots, N,$$

which are valid for arbitrary values of the independent variable x. This system of linear algebraic equations is used to determine the expansion coefficients $A_j(x)$, $A1_j(x)$ which are involved in the calculation of the second derivatives with respect to y on the k-th coordinate surface $\partial^2 u_k/\partial y^2_k$ and $\partial^2 c_k/\partial y^2_k$.

The system of nonlinear ordinary differential equations (8), (10)-(12) was solved by the Runge–Kutta method. Since $dP_1/d\theta$ is indeterminate, the right-hand sides of Eqs. (8), (10)-(12) were determined in two stages: 1) The pivotal coefficients were calculated and used to find the unknown boundary condition; 2) the right-hand sides were evaluated correctly. With this in mind Eqs. (8), (10)-(12) were reduced to the form

$$\frac{du_k}{d\theta} + L_k\frac{dp_k}{d\theta} + N_k\frac{dy_k}{d\theta} = R_k,$$
$$\frac{dy_k}{d\theta} - \frac{dy_{k-1}}{d\theta} + S_k\frac{du_k}{d\theta} + S_k\frac{du_{k-1}}{d\theta} = 0,$$
$$\frac{dp_k}{d\theta} - \frac{dp_{k-1}}{d\theta} + Q_k\frac{du_k}{d\theta} + T_k\frac{du_{k-1}}{d\theta} = \Omega_k, \quad k = 2, \ldots, N,$$
$$L_k = \frac{1}{u_k}, \quad N_k = \frac{1}{u_k}\left(\frac{9}{\mathrm{Re}}\theta\frac{\theta^2+1}{\theta^2+2} - \frac{\mathrm{E5\,E1}(\theta^2+2)}{(\theta^2+1)^{3/2}}u_k^2\right), \tag{13}$$
$$R_k = \frac{9}{\mathrm{E5\,E1\,Re}\,u_k}\frac{(\theta^2+1)^{3/2}}{(\theta^2+2)} + \frac{3\sqrt{\theta^2+1}}{\mathrm{E5\,E1\,Re}\,u_k}\frac{\partial^2 u_k}{\partial y_k^2}, \quad S_k = \frac{y_k - y_{k-1}}{u_k + u_{k-1}},$$

$$Q_k = \frac{y_k - y_{k-1}}{u_k + u_{k-1}}\left[\left(u_k^2 + u_{k-1}^2\right)\frac{\mathrm{E5\,E1}\,(\theta^2+2)}{2\,(\theta^2+1)^{3/2}} - \frac{9}{\mathrm{Re}}\,\theta\,\frac{\theta^2+1}{\theta^2+2} \mp\right.$$

$$\left.\mp\frac{3\mathrm{Ga}^{1/2}\mathrm{E5}^{1/2}\mathrm{E1}^2}{\mathrm{Re}}(u_k+u_{k-1})\right] - (y_k - y_{k-1})\left[u_k\frac{\mathrm{E5\,E1}\,(\theta^2+2)}{(\theta^2+1)^{3/2}} \mp \frac{3\mathrm{Ga}^{1/2}\mathrm{E5}^{1/2}\mathrm{E1}^2}{\mathrm{Re}}\right],$$

$$T_k = \frac{y_k - y_{k-1}}{u_k + u_{k-1}}\left[\left(u_k^2 + u_{k-1}^2\right)\frac{\mathrm{E5\,E1}\,(\theta^2+2)}{2\,(\theta^2+1)^{3/2}} - \frac{9}{\mathrm{Re}}\,\theta\,\frac{\theta^2+1}{\theta^2+2} \mp \frac{3\,\mathrm{Ga}^{1/2}\mathrm{E5}^{1/2}\mathrm{E1}^2}{\mathrm{Re}}\times\right.$$

$$\left.\times(u_k + u_{k-1})\right] - (y_k - y_{k-1})\left[u_{k-1}\frac{\mathrm{E5\,E1}\,(\theta^2+2)}{(\theta^2+1)^{3/2}} \mp \frac{3\,\mathrm{Ga}^{1/2}\,\mathrm{E5}^{1/2}\,\mathrm{E1}^2}{\mathrm{Re}}\right],$$

$$\Omega_k = (y_{k-1} - y_k)\left[\left(u_k^2 + u_{k-1}^2\right)\frac{\mathrm{E5\,E1}\,\theta\,(\theta^2+4)}{2\,(\theta^2+1)^{5/2}} + \frac{9}{\mathrm{Re}}\,\frac{\theta^4+5\theta^2+2}{(\theta^2+2)^2}\right].$$

We write the sought functions $du_k/d\theta$, $dy_k/d\theta$, $dp_k/d\theta$ in the form of pivotal relations:

$$du_k/d\theta = U_k + \hat{U}_k dp_1/d\theta,\quad dy_k/d\theta = Y_k + \hat{Y}_k dp_1/d\theta,$$
$$dp_k/d\theta = P_k + \hat{P}_k dp_1/d\theta.$$

Substituting these expressions into (13) and taking account of the fact that U_{k-1}, $\hat{U}_{k-1}$, Y_{k-1}, $\hat{Y}_{k-1}$, P_{k-1}, $\hat{P}_{k-1}$ are known, we obtain explicit expressions for the pivotal coefficients in the form of the following recurrence relations:

$$\begin{aligned}
U_k &= [R_k - N_k(Y_{k-1} - S_k U_{k-1}) - L_k(\Omega_k + P_{k-1} - T_k U_{k-1})]/(1 - L_k Q_k - N_k S_k),\\
\hat{U}_k &= [N_k(S_k \hat{U}_{k-1} - \hat{Y}_{k-1}) + L_k(T_k \hat{U}_{k-1} - \hat{P}_{k-1})]/(1 - L_k Q_k - N_k S_k),\\
Y_k &= -S_k U_{k-1} + Y_{k-1} - S_k U_k,\\
\hat{Y}_k &= -S_k \hat{U}_{k-1} + \hat{Y}_{k-1} - S_k \hat{U}_k,\\
P_k &= \Omega_k + P_{k-1} - T_k U_{k-1} - Q_k U_k,\\
\hat{P}_k &= \hat{P}_{k-1} - T_k \hat{U}_{k-1} - Q_k \hat{U}_k.
\end{aligned} \tag{14}$$

From the condition

$$dU_1/d\theta = dy_1/d\theta = dp_N/d\theta = 0 \tag{15}$$

and from Eqs. (13) we find the values of the pivotal coefficients for $k=2$:

$$U_2 = (R_2 - L_2\Omega_2)/(1 - L_2Q_2 - N_2S_2),\quad \hat{U}_2 = -L_2/(1 - L_2Q_2 - N_2S_2),$$
$$Y_2 = -S_2U_2,\quad \hat{Y}_2 = -S_2\hat{U}_2,\quad P_2 = \Omega_2 - Q_2U_2,\quad \hat{P}_2 = 1 - Q_2\hat{U}_2.$$

Then using Eqs. (14) we find the values of the pivotal coefficients for any k. In particular for $k=N$ we have $dp_N/d\theta = P_N + \hat{P}_N dp_1/d\theta$, but $dp_N/d\theta$ is known from (15) and, consequently, $dp_1/d\theta = (dp_N/d\theta - P_N)/\hat{P}_N$. Then by a reverse pivotal we calculate the values of the right-hand sides of the system of differential equations (13).

After determining the velocity field u_k the system of differential equations was solved for the concentration in the liquid layer by the Runge–Kutta method.

The mass-transfer coefficient in the liquid film was calculated from the expression

$$D\left(\frac{\partial c}{\partial y}\right)_{y=y_N} = \frac{d}{dx}\int_0^{y_N} ucdy, \tag{16}$$

which was obtained after integrating the convective diffusion equation across a liquid film of variable thickness and using (6) at the interface.

Averaging Eq. (16) with respect to the longitudinal coordinate over a portion with a characteristic length L we obtain

$$\frac{1}{L}\int_0^L D\left(\frac{\partial c}{\partial y}\right)_{y=y_N} dx = \frac{1}{L}\int_0^L\left[\frac{d}{dx}\int_0^{y_N} ucdy\right]dx. \tag{17}$$

After averaging both sides of Eq. (17) over the surface of contact, evaluating the integral using the boundary condition (5), and introducing the dimensionless variable $x = \delta_p\,\mathrm{Re}\,\mathrm{Pr}\bar{x}$, we obtain an expression for the average mass-transfer coefficient in the liquid film:

$$\beta = \frac{1}{Lc_p}\int_0^L D\left(\frac{\partial c}{\partial y}\right)_{y=y_N} dx = \frac{1}{Lc_p}\left[\int_0^{y_N} ucdy\right]_{x=L} = \frac{u_p\delta_p}{L}\left[\int_0^{\bar{y}_N} \bar{u}\bar{c}d\bar{y}\right]_{\bar{x}=\frac{L}{\delta_p \mathrm{Re}\,\mathrm{Pr}}} = \frac{u_p\bar{\delta}_p}{L}HT\Bigg|_{\bar{x}=\frac{L}{\delta_p \mathrm{Re}\,\mathrm{Pr}}}.$$

The algorithm described above was used to calculate the velocity field, concentrations, surface of separation, and tangential stress on the wall of the spiral channel in the entrance region as a function of the slit width, the Reynolds number, and the dimensionless characteristic of the spiral E5.

Figure 1 shows the characteristic form of the development of the velocity profile in the liquid film at various cross sections for Re = 100, E5 = 0.1, and E1 = 1/3. Figure 2 shows the characteristic dependence of the local tangential stress τ on the dimensionless length of the spiral for Re = 300, E5 = 0.1; 1) E1 = 0.1; 2) 1/7; 3) 1/3; 4) 2/3; 5) 1; 6) 2. The tangential stress varies sharply in the region near the edge of the film distributor; for $\delta_p/h_0 > 1$ it decreases near the edge of the film distributor and for $\delta_p/h_0 < 1$ it increases. Figure 3 shows how the ratio of the local tangential stress to the tangential stress in the stabilized region depends on the dimensionless length of the spiral for Re = 300, E5 = 0.1; 1) E1 = 0.1; 2) 1/7; 3) 1/3; 4) 2/3; 5) 1. The ratio τ/τ_p for $\delta_p/h_0 < 1$ increases with the distance from the edge of the film distributor. The dimensionless distance $\bar{x}$ at which the ratio τ/τ_p approaches unity depends on E1, increasing as E1 decreases.

A comparison of the results of a calculation of the velocity and tangential stress in a liquid film running down a vertical channel by gravity [3] and a film on a rotating Archimedes spiral shows that these quantities develop in the same way.

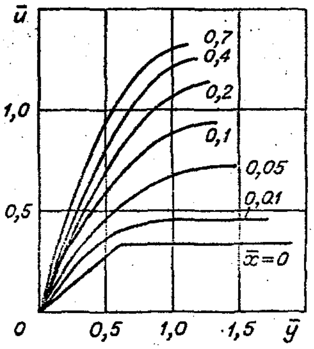

Fig. 1

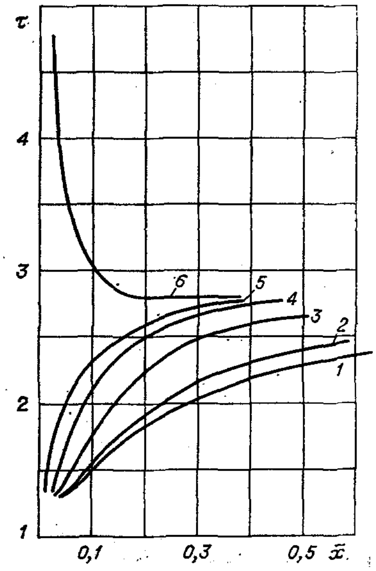

Fig. 2

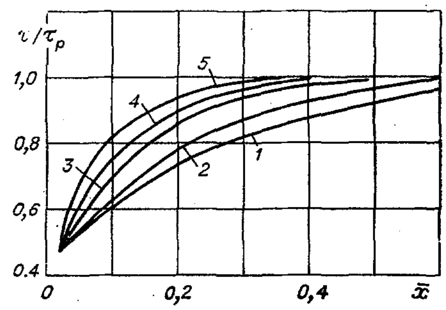

Fig. 3

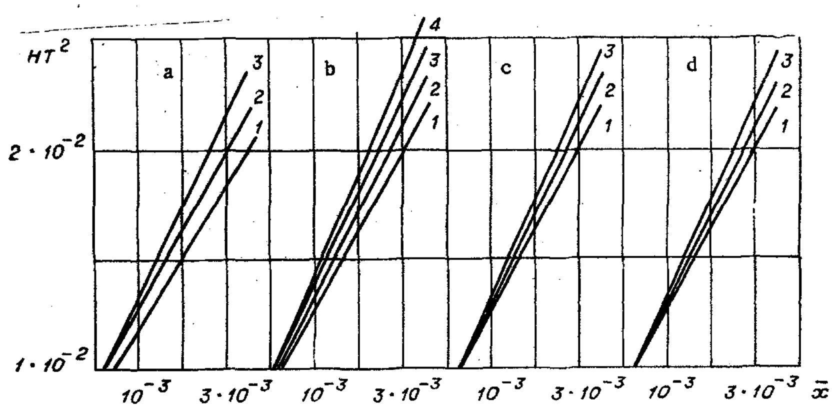

Fig. 4

Figure 4 shows the characteristic form of the dependence of HT^2 on the dimensionless length of the spiral for a) Re = 100, E5 = 0.1, E1 = 0.1 and 1) Pr = 100, 2) 300, 3) 1000; b) Re = 100, E5 = 0.1, Pr = 300 and 1) E1 = 0.1, 2) 0.4, 3) 1, 4) 1.6; c) Re = 100, E1 = 0.1, Pr = 300 and 1) E5 = 0.1, 2) 0.5, 3) 1; d) E5 = 0.1, Pr = 300, E1 = 0.1 and 1) Re = 100, 2) 500, 3) 1000. It is clear that large values of HT^2 correspond to large values of E1, E5, Re, and Pr. These parameters have little effect on the intercept on the HT^2 axis which varies from $0.85 \cdot 10^{-2}$ to 10^{-2} and can be taken as 0.009.

Over the range of parameters investigated HT^2 can be approximated to within about 5% by the expression

$$HT^2 = (3.5 + 0.7\mathrm{E1} + 1.2\mathrm{E5} + 0.001\mathrm{Re} + 0.0007\mathrm{Pr})\bar{x} + 0.009, \tag{18}$$

Taking account of (18) the expression for the average mass-transfer coefficient in the liquid phase can be written in the form

$$\beta = \frac{u_p^{1/2} D^{1/2}}{L^{1/2} 3^{1/2}} \sqrt{3.5 + 0.7\,\mathrm{E1} + 1.2\,\mathrm{E5} + 0.001\,\mathrm{Re} + 0.0007\,\mathrm{Pr} + 0.009 \frac{\delta_p \mathrm{Re\,Pr}}{L}}.$$

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